Fixing Shin's Reading Algorithm for Peirce's Existential Graphs

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Abstract. In her book "The Iconic Logic of Peirce's Graphs", S. J. Shin elaborates the diagrammatic logic of Peirce's Existential Graphs. Particularly, she provides translations from Existential Graphs to first order logic. Unfortunately, her translation is not in all cases correct. In this paper, the translation is fixed by means of so-called *single object ligatures*.

1 Introduction

The well-known system of Existential Graphs (EGs) by Peirce is divided into three parts which are called *Alpha*, *Beta* and *Gamma* which are built upon each other. The step from Alpha to Beta corresponds to the step from propositional logic to first order logic (FOL). In this step, a new syntactical element, the *line of identity (LoI)*, is added to EGs. LoIs are used to denote both the existence of objects and the identity between objects, and they are represented as heavily drawn lines. LoIs which are sometimes assembled to networks termed *ligatures*. Consider the four EGs of Fig. 1.

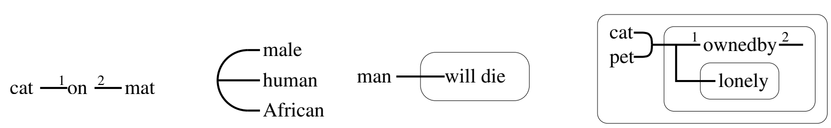


Fig. 1. Four Peirce graphs with so-called single-object-ligatures

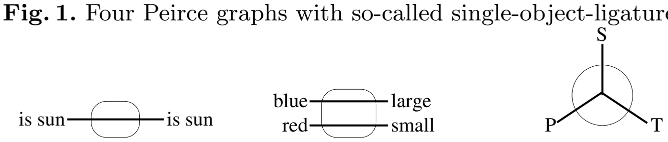


Fig. 2. Three Peirce graphs with non-single-object ligatures

The meaning of the graphs of Fig. 1 is 'a cat is on a mat', 'there exists a male, human african', 'there exists a man who will not die', and 'every pet cat is owned by someone and is not lonely'. In all these graphs, LoIs and ligatures, even if they cross cuts, are used to denote a single object. To put it more formally: We can provide translations of the EGs to formulas of FOL where we assign to each ligature one variable. In fact, we can translate the EGs of Fig. 1 to FOL as follows:

1. $\exists x.\exists y.(cat(x) \wedge on(x,y) \wedge mat(y))$
2. $\exists x.(male(x) \wedge human(x) \wedge African(x))$
3. $\exists x.(man(x) \wedge \neg willdie(x))$
4. $\neg\exists x.(cat(x) \wedge pet(x) \wedge \neg(\exists y : ownedby(x,y) \wedge \neg lonely(x)))$

But the EGs of Fig. 2 show that this reading of ligatures is not always that simple: A ligature may stand for more than one object. For example, the first graph reads 'there are at least two suns'. Thus, in the translations of these graphs, we have to assign more than one variable to the ligatures. They are:

1. $\exists x.\exists y.(issun(x) \wedge issun(y) \wedge x \neq y)$
2. $\exists x.\exists y.\exists u.\exists v.(blue(x) \wedge large(y) \wedge red(u) \wedge small(v) \wedge \neg(x = y \wedge u = v))$
3. $\exists x.\exists y.\exists z.(S(x) \wedge P(y) \wedge T(z) \wedge \neg(x = y \wedge y = z))$

In [8], Shin thoroughly elaborates the diagrammatic logic of Peirce's EGs. But is has to be said that from a mathematical point of view, her treatise lacks preciseness (see [1] for a thorough discussion). Moreover, her elaboration unfortunately contains some flaws. Particularly, in her translation from EGs to FOL-formulas, she assigns in some cases too few variables to ligatures, which sometimes yields wrong formulas. In the next section, we clarify the terms "line of identity" and "ligature", and moreover, the notion of so-called "single object ligature" is introduced. The next section briefly discusses the flaw in Shin's translation, and then uses single object ligature to fix it. Further flaws in [8] can be found in her transformation rules. This will be briefly discussed in the outlook.

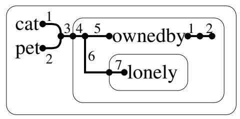
2 Lines of Identity and Ligatures

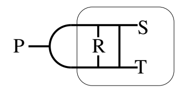
Peirce describes a LoI as follows: '*The line of identity is [...] a heavy line with two ends and without other topical singularity (such as a point of branching or a node), not in contact with any other sign except at its extremities.*' (4.116: We use the common notation to refer to [5]). It is important to note that LoIs do are neither allowed to branch nor they allowed to *cross* cuts. But it is allowed that they *touch* other elements at their extremities, i.e.:

1. Two or three LoIs may be connected at their endpoints. If three LoIs are connected, the point where they meet is a *branching point*.
2. LoIs may end on a cut. Particularly, it is allowed that LoIs are connected directly on a cut. Due to this possibility, we can have heavily drawn lines (composed of several LoIs) which cross a cut.

LoIs may be assembled to connected networks termed LIGATURES. Peirce writes in 4.407: '*A collection composed of any line of identity together with all others that are connected with it directly or through still others is termed a ligature. Thus, ligatures often cross cuts,[...]*', and in 4.416, he writes '*The totality of all the lines of identity that join one another is termed a ligature.* ' So he explicit discriminates between one line of identity and a linked structure of lines of identity called ligature. Particularly, each LoI is a ligature, but not vice versa.

Consider the last graph of Fig. 1. This graph has two maximal ligatures. The left one is composed of (at least) seven LoIs. In the diagram below, these LoIs are numbered, and all endpoints of LoIs are indicated as bold spots. The right heavy line is a single LoI, but even single LoIs can be understood to be composed of smaller LoIs as well. This is indicated by breaking up this line into two LoIs.



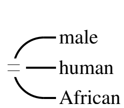


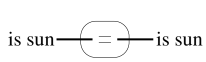
As Peirce terms the *totality* of all the LoIs that join one another a ligature, Peirce's understanding of a ligature is a *maximal* connected network of LoIs. To clarify matters, in this paper *each* connected network of LoIs is called ligature.

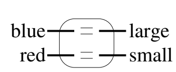
In all graphs of Fig. 2, a part of the ligature traverses a cut (i.e., there is a cut c and a heavily drawn line l which is part of the ligature such that both endpoints of l are placed on c and the remainder of l is enclosed by c). Such a device denotes non-identity of the endpoints of l, so a ligature containing such a device l usually denotes different objects. But if such a device does not occur, it has been shown in [2] that the ligature denotes a single object. For this reason, a ligature L such that no part of L traverses any cut will be called SINGLE-OBJECT-LIGATURE (SO-LIGATURE). Note that so-ligatures may contain cycles or may cross a cut more than once: An example for this is the right graph above.

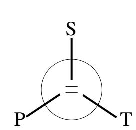
3 Fixing Shin's Reading of Graphs

The clue to read arbitrary EGs is to break up non-so-ligatures into several so-ligatures by adding additional equality relations. Peirce writes in [6] that the second graph of Fig. 1 '*is a graph instance composed of instances of three indivisible graphs which assert 'there is a male', 'there is something human' and 'there is an African'. The syntactic junction or point of teridentity asserts the identity of something denoted by all three.*' That is, we can replace the branching point by a relation $\doteq_3$, termed TERIDENTITY, expressing that the objects denoted by the attached LoIs are all identical. The corresponding graph is the leftmost graph below (the index 3 on the relation-sign is omitted). Even simpler, when two LoIs meet in a point, we can replace this point by the usual binary identity relation $\doteq_2$. So, if an EG with a non-so-ligature is given, we can replace some branching points by $\doteq_3$ and some non-branching points by $\doteq_2$ until the non-so-ligature is split up into several so-ligatures. The graphs which correspond to the graphs of Fig. 2 are the second, third, and fourth graphs shown below.



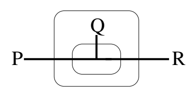






Now we have two, four and three so-ligatures, respectively, that is why the above translations of these graphs to FOL need two, four and three variables.

Replacing branching points by a ternary identity relation has already been carried out by Zeman in [9] in his translation. But Zeman replaces *each* branching point by an identity relation, and moreover, he *always* splits heavily drawn lines if they cross a cut more than once (by adding the dyadic identity). Thus in nearly all cases, Zeman translation uses far more variables than necessary, and the resulting formulas are hard to read. This has been thoroughly discussed by Shin in [8]. She correctly points out that Zeman's reading algorithm for existential graphs is comprehensive and yields correct results (in contrast to Robert's reading in [7], as she argues), but it usually yields a '*translation that looks more complicated than the original graph*'. A main reason for her criticism is the '*mismatch between the number of lines in a graph and the number of the variables in the translation*'. Shin tries to overcome this problem in her reading algorithm, but unfortunately, sometimes she assigns *too few* variables to ligatures. To see this, consider the following graph $\mathfrak{G}_z$ and the model $\mathcal{M}$ with two objects a, b:

$\mathfrak{G}_z :=$ 

$\mathcal{M} :=$

	P	Q	R
a	×		
b			×

Now we have the following readings of $\mathfrak{G}_z$:

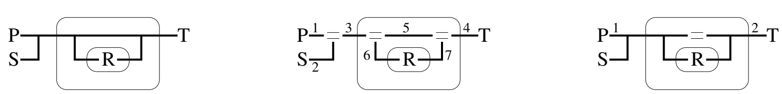
1. Zeman: $f_z := \exists x.\exists z.(Px \wedge Rz \wedge \neg\exists y.(Qy \wedge \neg(x = y \wedge y = z)))$
2. Shin: $f_s := \exists x.(Px \wedge \forall y.(x = y \vee \neg Qy) \wedge Rx)$ (see p. 128 of [8])

Note that f_z needs three variables, but f_s needs only two variables. We obviously have $f_s \models f_z$. But we have $\mathcal{M} \models f_z$ and $\mathcal{M} \not\models f_s$, thus $f_z \not\models f_s$. So, f_z is a strictly weaker formula than f_s. Particularly, Shin's reading is not correct.

With the observation that so-ligatures denote single objects, we can now fix Shin's reading of EGs in order to obtain a correct translation of EGs to formulas which, as Shin writes, '*respects Peirce's intuition about his graphical system*'. Shin's algorithm is provided in [8] on page 122ff. The first steps of Shin's algorithm have to be replaced by the following instructions:

1. Transform all ligs. into so-ligs. by appropriately adding relation signs '$\doteq$'
2. Erase all double cuts
3. Assign a variable to the outermost part of each so-ligature.
4. Continue with step 3. of Shin's reading algorithm.

Below, a sample graph for Zeman's and Shin's improved reading is provided. In the middle, the ligatures of the graph are split due to Zeman's algorithm, on the right, this is done according to Shin's improved algorithm. The corresponding translations f_Z of Zeman and f_S of Shin/Dau show the difference.



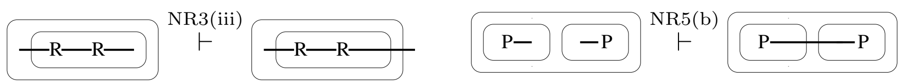
$$f_Z = \exists x_1, x_2, x_3, x_4 : [P(x_1) \wedge S(x_2) \wedge x_1 = x_2 \wedge x_2 = x_3 \wedge T(x_4) \wedge \neg\exists x_5, x_6, x_7 : (x_3 = x_6 \wedge x_6 = x_5 \wedge x_5 = x_7 \wedge x_7 = x_4 \wedge \neg R(x_6, x_7))]$$

$$f_S = \exists x_1, x_2 : [P(x_1) \wedge S(x_1) \wedge T(x_2) \wedge (x_1 \neq x_2 \vee R(x_1, x_2))]$$

4 Future Research

Shin argues in [8] very clearly that have to evaluate EGs in terms of diagrammatic instead of symbolic systems. The syntax, the reading, and the calculus, has to respect the visual features of EGs.

Ligatures are a distinguishing element of EGs, thus we particularly need a precise understanding of them. As we have seen, even an expert as Shin sometimes fails in their handling. This unfortunately holds true for her transformation rules as well. For example, her rule NR3(iii) allows to extend a loose end of an LoI outwards from an even to an even area, if some restrictions are fulfilled. Below, a correct application of the rule is exemplified. Her rule NR5(b) allows to connect '*tokens of the same type*'. This rule is exemplified as well. But in both examples, there are models in which the left graph is true, but the right is not. So these rules (and thus their counterpart NR4(iii) and NR6(a)) are *not sound*.



In [2], a mathematical elaboration of EGs, particularly of ligatures, is provided, and the results of [2] have been used in this paper to fix Shin's reading of EGs. I wholeheartly agree to Shin's approach that EGs should be redesigned in order to fully implement all their iconic aspects. So the next step in developing EGs as diagrammatic, efficient and formal logic system is to fix and formalize Shin's transformation rules in a way that Shin's intention is best reflected as possible.

References

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